Statistics

Lecture 15

Feb 19-8:47 AM

Class Quiz 6

Consider a binomial prob. dist. with

$n = 80$ and $P = .6$

1) Find $P(x \leq 50)$

$= \text{binomcdf}(80, .6, 50)$

$= \boxed{.714}$

2) Find $P(x \geq 45)$

$= 1 - P(x \leq 44)$

$= 1 - \text{binomcdf}(80, .6, 44)$

$= \boxed{.789}$

3) Find its mean.

$\mu = np$

$= 80(.6) = \boxed{48}$

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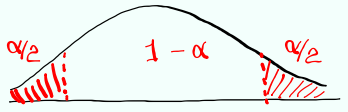
α Alpha

$0 < \alpha < 1$

$\frac{\alpha}{2}$ is the area on each tail of the graph of Prob. dist.

$1-\alpha$ is the middle area.



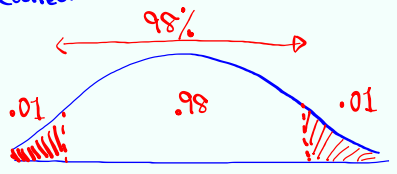
α is called the Significance level.

$1-\alpha$ in % is called the Confidence level.

ex: for $\alpha = .02$

$\frac{\alpha}{2} = .01$

$1-\alpha = .98 = 98\%$



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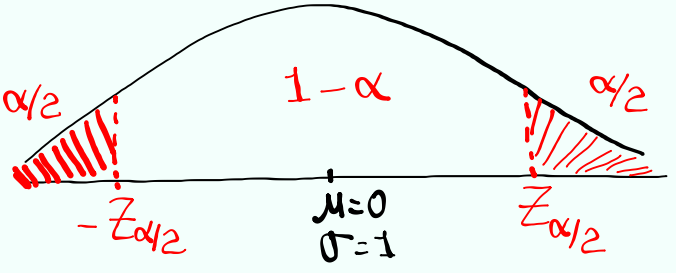
$Z_{\alpha/2}$ is the critical value with right-tail area of $\alpha/2$ of the standard normal distribution curve. $(\mu=0, \sigma=1)$



To find $Z_{\alpha/2}$ ⇒ use invNorm (Left Area, μ, σ)

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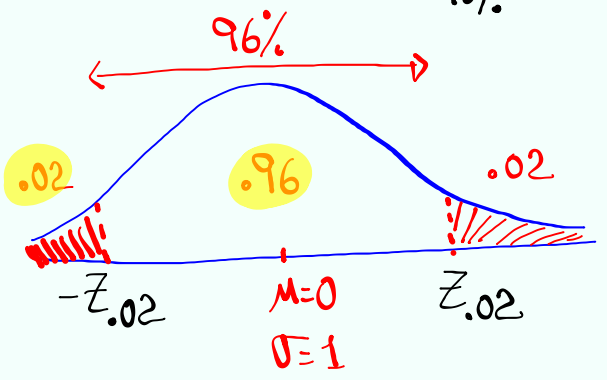
Find $z_{.02}$

Confidence level 96%.

$\frac{\alpha}{2} = .02$

$\alpha = .04$

$1 - \alpha = .96 = 96\%$



$z_{.02} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

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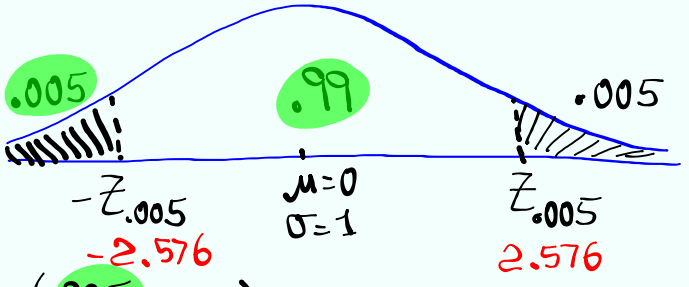
Find $\pm z_{\alpha/2}$ for $\alpha = .01$.

$\alpha = .01$ Significance level

$1 - \alpha = .99 = 99\%$ Confidence level (Middle area)

$\frac{\alpha}{2} = .005$ area on each tail



$z_{.005} = \text{invNorm}(.995, 0, 1) = 2.576$

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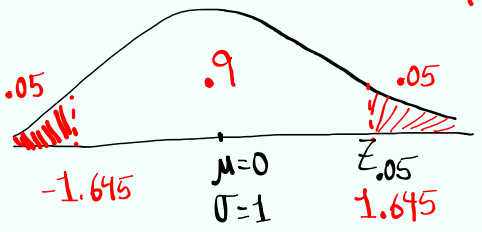
Find $z_{\alpha/2}$ for 90% Confidence level.

Middle area .9



$1-\alpha = .9$

$\alpha = .1$

$\frac{\alpha}{2} = .05$

$z_{.05} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

when α not given $\Rightarrow$ use .05

when Conf. level not given $\Rightarrow$ use 95%

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t - Dist.

It is bell-shape, symmetric with total area 1.

$\mu = 0$ but σ unknown

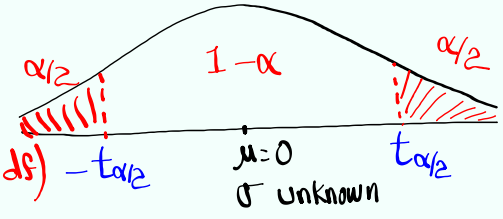
It comes with degrees of freedom.

$t_{\alpha/2}$ Right-Tail area is $\alpha/2$

Left-Tail area is $\alpha/2$

Middle area is $1-\alpha$.

we use invT Command



invT(Left area, df) $-t_{\alpha/2}$

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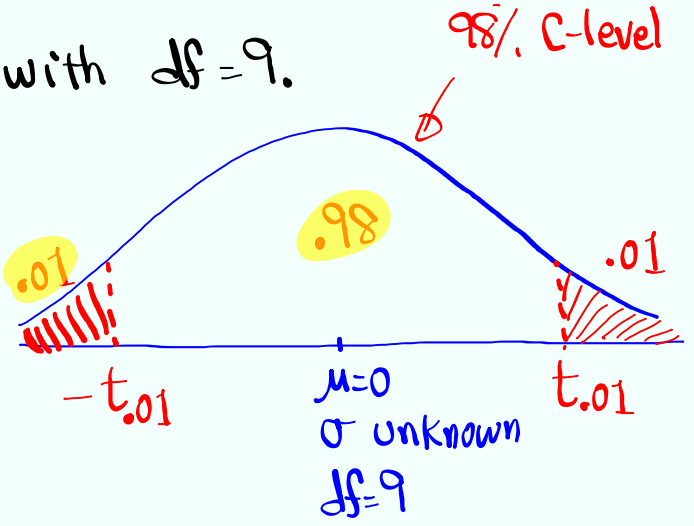
Find $t_{.01}$ with $df = 9$.

$\frac{\alpha}{2} = .01$

$\alpha = .02$

$1 - \alpha = .98$



$t_{.01} = invT(.99, 9) = 2.821$

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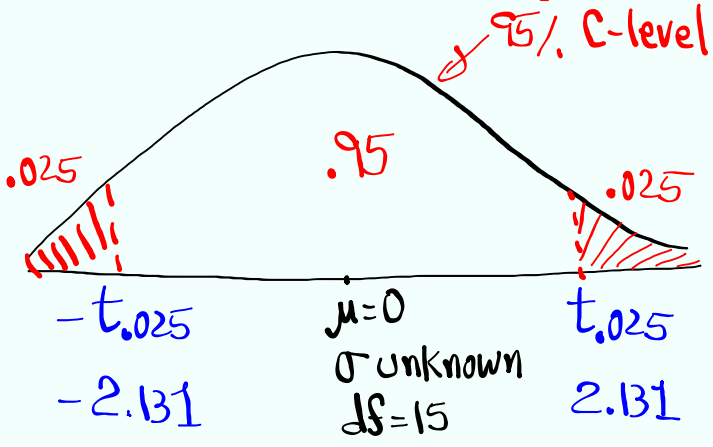
Find $\pm t_{\alpha/2}$ for $\alpha = .05$ with $df = 15$.

$\alpha = .05$

$\frac{\alpha}{2} = .025$

$1 - \alpha = .95$



$t_{.025} = invT(.975, 15) = 2.131$

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Find $t_{\alpha/2}$ for 96% C-level with df=29.

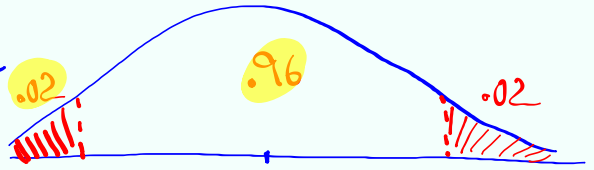
96% C-level

Middle area .96

$1-\alpha = .96$

$\alpha = .04$

$\alpha/2 = .02$



$-t_{.02}$ $\quad \mu = 0$ $\quad t_{.02}$

-2.150 $\quad$ σ unknown $\quad 2.150$

df = 29

$t_{.02} = \text{INVT}(.98, 29) = 2.150$

As df gets bigger and bigger,

$$t_{\alpha/2} \approx Z_{\alpha/2}$$

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What is degrees of freedom?

It gets determined by topics.

Non-Statistical Examples.

15 Students, I bring 15 donuts.

You can have only one donut.

Mariah	15 choices
Isabel	14 "
Brisa	13 "
⋮	⋮
Darren	1 Donut left (No choice)

df = 14

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You did your laundry. You have 7 shirts.

Monday 7 clean shirts

Tuesday 6 " "

Wednesday 5 " "

⋮ ⋮ ⋮

Sunday 1 clean shirt.

$df = 6$

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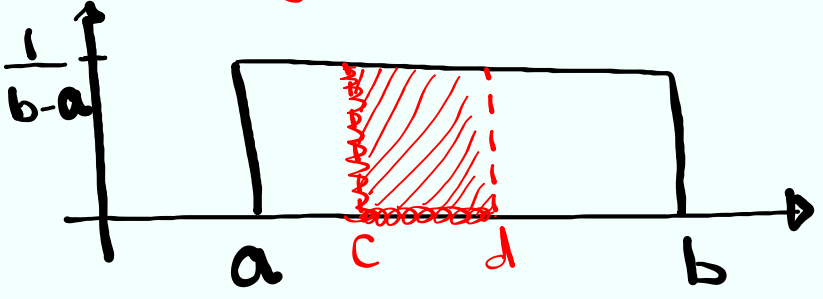
Uniform Prob. dist. for all values from a to b.

Graph is rectangular. Total Area = 1.



$P(x=c)=0$ $P(c<x<d)=(d-c)\cdot\frac{1}{b-a}$

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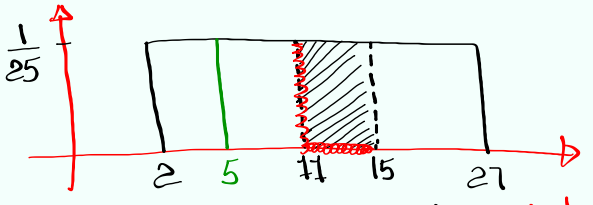
Consider a uniform prob. dist. for all values from 2 to 27.



1) $P(x=5)=0$

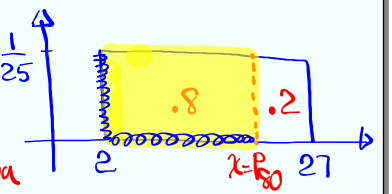
2) $P(11<x<15)=(15-11)\cdot\frac{1}{25}$

$=\boxed{\frac{4}{25}}$

3) Find $x=P_{80}$

80% below — Left area .8

20% above — Right area .2



$(x-2)\cdot\frac{1}{25}=.8$

$x-2=25(.8)$

$x-2=20$

$\boxed{x=22}$

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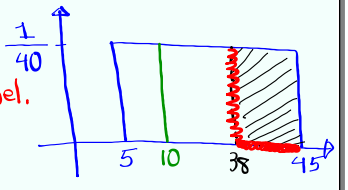
Consider a uniform prob. dist. for all values from 5 to 45.

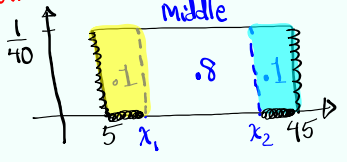


1) Draw & clearly label.

2) $P(x=10)=0$

3) $P(x>38)=(45-38)\cdot\frac{1}{40}=\boxed{\frac{7}{40}}$

4) Find two values that separate the middle 80% from the rest.



$(x_1-5)\cdot\frac{1}{40}=.1$

$x_1-5=40(.1)$

$x_1-5=4$

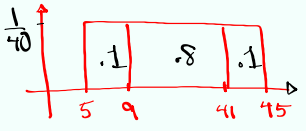
$\boxed{x_1=9}$

$(45-x_2)\cdot\frac{1}{40}=.1$

$45-x_2=40(.1)$

$45-x_2=4$

$\boxed{x_2=41}$



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Standard Normal Prob. Dist.

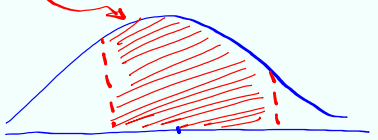
1) use Z, $P(Z=c)=0$

2) Graph has a bell-shape, symmetric with total area 1.

3) Mean = Mode = Median

4) $\mu=0$, $\sigma=1$

5) $P(a<Z<b)$ is the Corresponding area within the Curve.



How to find it

2nd VARS

normalcdf(Lower, Upper, μ, σ)

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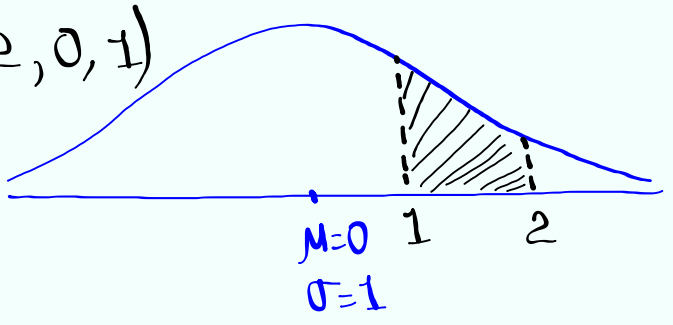
$P(1<Z<2)$

= normalcdf(1, 2, 0, 1)

= .136

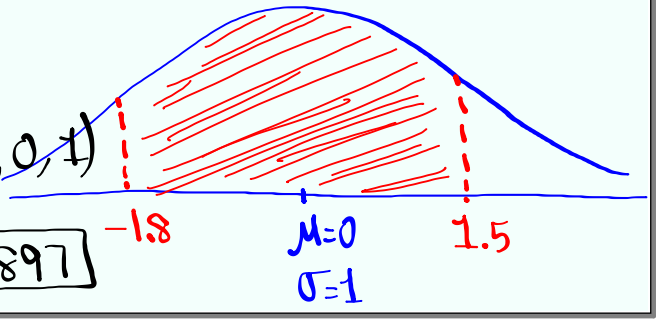


$P(-1.8<Z<1.5)$

= normalcdf(−1.8, 1.5, 0, 1)

(−)

= .897



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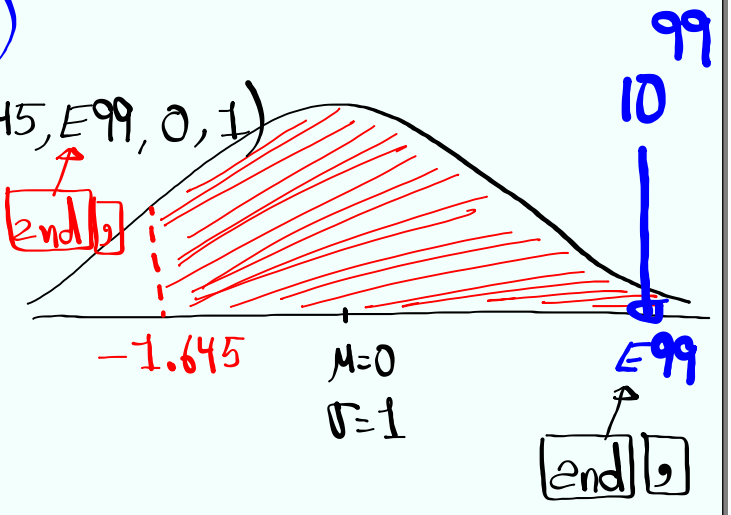
$P(Z > -1.645)$

= normalcdf(−1.645, E99, 0, 1)

(−) 2nd [2]

= .950



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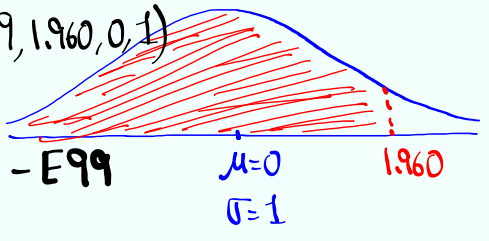
$P(Z < 1.960)$

= normalcdf(−E99, 1.960, 0, 1)

(−) 2nd [2]

= .975

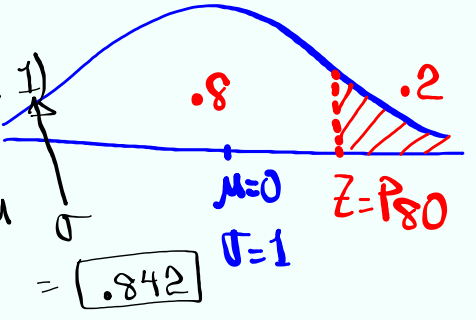


Find $Z = P_{80}$

invNorm(.8, 0, 1)

left Area, μ, σ

= .842



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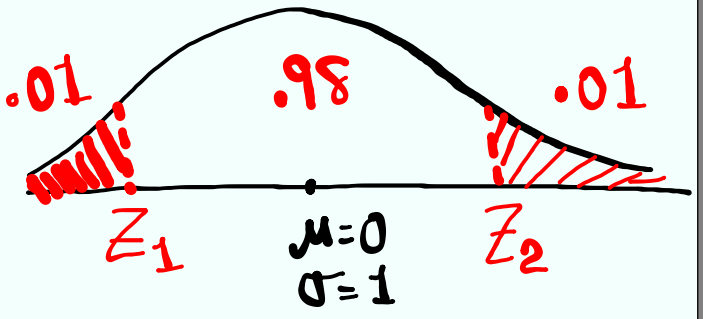
Find two z-values that separate the middle 98% from the rest.



$z_1 = \text{invNorm}(.01, 0, 1) = \boxed{-2.326}$

$z_2 = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

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